Construction of a PDA

Lecture - 22

Q) Obtain a PDA to accept the lang $L(M) = \{wcw^R \mid w \in (a+b)^*\}$ where w^R is reverse of w

$w = abb$

$L = \{abbcbba\}$ palindrome.

(push onto stack)

once c is seen, change state, from here remove symbols from stack

- stack empty $\Rightarrow$ string is a palindrome.

Step 1 Input symbols can be a or b.

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$ } empty stack

$\delta(q_0, b, Z_0) = (q_0, bZ_0)$ }

$\delta(q_0, a, a) = (q_0, aa)$ }

$\delta(q_0, b, a) = (q_0, ba)$ } one symbol was already on stack.

$\delta(q_0, a, b) = (q_0, ab)$ }

$\delta(q_0, b, b) = (q_0, bb)$ }

Step 2 Input symbol is c.

$\delta(q_0, c, Z_0) = (q_1, Z_0)$

$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_0, c, b) = (q_1, b)$

Step 3 Input symbol can be a or b.

$\delta(q_1, a, a) = (q_1, \varepsilon)$

$\delta(q_1, b, b) = (q_1, \varepsilon)$

Step 4 In state q_1, if string is a palindrome then there is no input symbol to be scanned and the stack is also empty i.e, stack contains Z_0.

$\therefore \delta(q_1, \varepsilon, Z_0) = (q_2, Z_0)$

and PDA has accepted the language

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$\therefore Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{a, b, Z_0\}$

δ is as shown below

List of the above transitions here

$a, Z_0 \mid aZ_0$

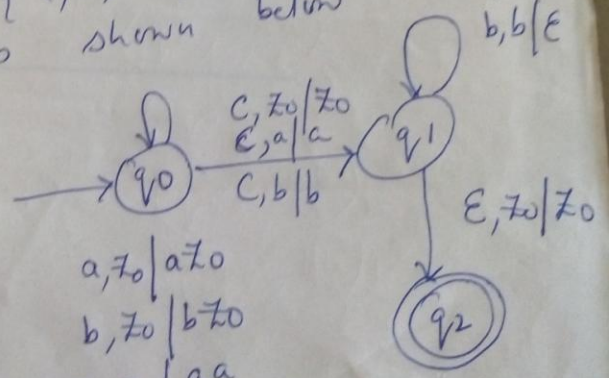
$b, Z_0 \mid bZ_0$

$a, a \mid aa$

$b, a \mid ba$

$a, b \mid ab$

$b, b \mid bb$



$q_0 \in Q$ is the start

$Z_0 \in \Gamma$ is initial symbol of stack

$F = \{q_2\}$ is the final state

To accept the string (sequence of moves) (3)

Initial ID:

$q_0, aabCbaa, Z_0 \vdash (q_0, abCbaa, aZ_0)$

$\vdash (q_0, bCbaa, aaZ_0)$

$\vdash (q_0, Cbaa, baaZ_0)$

$\vdash (q_1, baa, baaZ_0)$

$\vdash (q_1, aa, aaZ_0)$

$\vdash (q_1, a, aZ_0)$

$\vdash (q_1, \varepsilon, Z_0)$

$\vdash (q_2, \varepsilon, Z_0)$

final configuration.

try $(q_0, aabCbab, Z_0)$ find if it is rejected.

1) Obtain a pda to accept the language (4)
$L = \{a^n b^n / n \geq 1\}$ by a final state

Sol: n # of a's followed by n # of b's

Step 1 Input is a

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

Step 2 Input is b

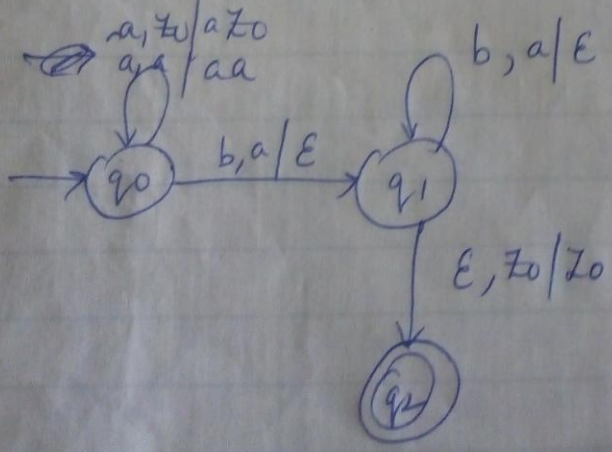
$\delta(q_0, b, a) = (q_1, \varepsilon)$

Step 3 once in q_1, rest of the symbols to be scanned will be only b's, and for each b there should be a corresponding a on the stack.

$\delta(q_1, b, a) = (q_1, \varepsilon)$

Step 4

$\delta(q_1, \varepsilon, Z_0) = (q_2, Z_0)$



ccept a string

$(q_0, aaabbb, z_0) \vdash (q_0, aabbb, az_0)$
$\vdash (q_0, abbb, aaz_0)$
$\vdash (q_0, bbb, aaaz_0)$
$\vdash (q_1, bb, aaz_0)$
$\vdash (q_1, b, az_0)$
$\vdash (q_1, \varepsilon, z_0)$
$\vdash (q_2, \varepsilon, z_0)$
final configuration

Try: aabbb

~~Step 2~~ $L = \{a^n b^{2n} \mid n \geq 1\}$

Step 1 $\delta(q_0, a, Z_0) = (q_0, aaZ_0)$

$\delta(q_0, a, a) = (q_0, aaa)$

for every a push aa onto a stack.

Step 2.

$\delta(q_0, b, a) = (q_1, \varepsilon)$

Step 3. remove all the symbols by taking a for every b.

$\delta(q_1, b, a) = (q_1, \varepsilon)$

Step 4.

$\delta(q_1, \varepsilon, Z_0) = (q_2, \varepsilon)$.

$(q_0, aabbbb, Z_0) \vdash (q_0, abbbb, aaZ_0)$

$\vdash (q_0, bbbb, aaaaZ_0)$

$\vdash (q_1, bbb, aaaZ_0)$

$\vdash (q_1, bb, aaZ_0)$

$\vdash (q_1, b, aZ_0)$

$\vdash (q_1, \varepsilon, Z_0)$

$\vdash (q_2, \varepsilon, Z_0)$. final cup

to accept balanced parentheses (7).

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$\delta(q_0, (, Z_0) = (q_1, (Z_0)$

$\delta(q_0, [, Z_0) = (q_1, [Z_0)$

step 1 q_0 is the accepting state (an empty string)

step 2 $\delta(q_0, (, Z_0) = (q_1, (Z_0)$

$\delta(q_0, [, Z_0) = (q_1, [Z_0)$

step 3 $\delta(q_1, (, () = (q_1, (()$

$\delta(q_1, [, () = (q_1, [()$

$\delta(q_1, (, [) = (q_1, ([)$

$\delta(q_1, [, [) = (q_1, [[)$

step 4 $\delta(q_1,), () = (q_1, \varepsilon)$

$\delta(q_1,], [) = (q_1, \varepsilon)$

step 5 $\delta(q_1, \varepsilon, Z_0) = (q_0, Z_0)$

↑ final state.

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